Maximize the Volume of a Cylinder Given Its Surface Area

• For a given surface area, the cylinder with maximum volume has a height equal to its diameter.

• That is, \( h = d \) or \( h = 2r \).

The front view of this cylinder is a square.

To Maximize The Volume of a Cylinder Given Surface Area:

1. **Substitute** the given surface area *AND* \( 2r \) for \( h \) in the Surface Area formula for a cylinder and solve for \( r \).

   ⇒ *The result will be the radius of a cylinder with maximum volume.*

2. **Substitute** the radius to find the maximum volume of the cylinder.
**Example:**

Determine the dimensions of the cylinder with maximum volume that can be made 600m$^2$ of aluminum. Then, find its volume.

1. **Solve for $r$:** Substitute the given Surface Area AND $2r$ for $h$ in the Surface Area formula.

\[
SA = 2\pi r^2 + 2\pi rh \\
600 = 2(3.14)r^2 + 2(3.14)r(2r) \\
600 = 6.28r^2 + 12.56r^2 \\
600 = 18.84r^2 \\
31.85 = r^2 \\
\sqrt{31.85} = r \\
5.64 = r
\]

- The radius of the can should be 5.64 cm.
- The height is twice the value of the radius or 11.28 cm.
2. Substitute the radius in the volume formula and calculate the MAXIMUM volume.

\[ V = 2\pi r^2 h \]

\[ = 2(3.14)(5.64)^2 \]

\[ = 199.76 \text{ cm}^3 \]

**Minimize the Surface Area of a Cylinder Given Its Volume**

• For a given volume, the cylinder with minimum surface area has a height equal to its diameter.

• That is, \( h = d \) or \( h = 2r \).

The front view of this cylinder is a square.
To Minimize The Surface Area of a Cylinder Given Volume:

1. Substitute the given volume AND $2r$ for $h$ in the Volume formula for a cylinder and solve for $r$.

   ⇒ The result will be the radius of a cylinder with minimum surface area.

2. Substitute the radius to find the minimum surface area of the cylinder.

Example:

Determine the least amount of aluminum required to construct a cylinder can with a 1-L capacity.

$1 \text{ L} = 1000 \text{ cm}^3$
1. **Solve for** $r$: Substitute the Volume 1L (1000 cm$^3$) **AND** $2r$ for $h$ in the Volume formula.

\[ V = \pi r^2 h \]
\[ 1000 = (3.14)r^2(2r) \]
\[ 1000 = (3.14)(2r^3) \]
\[ 1000 = 6.28r^3 \]
\[ 159.24 = r^3 \]
\[ \sqrt[3]{159.24} = r \]
\[ 5.42 = r \]

- The radius of the can should be 5.42 cm.
- The height is twice the value of the radius or 10.84 cm.

2. To find the **MINIMUM** amount of aluminum required, substitute the radius and calculate the surface area.

\[ SA = 2\pi r^2 + 2\pi rh \]
\[ = 2(3.14)(5.42)^2 + 2(3.14)(5.42)(10.84) \]
\[ = 664 \text{ cm}^2 \]